



# SAV Assembly 2013

## One-year insurance risk for non-life business

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**Audit.Tax.Consulting.Corporate Finance.**

# One-year insurance risk for non-life business

## Agenda

- Introduction
- Decomposition of the one-year risk
- One-year reserve risk
- One-year premium risk
- Parameter uncertainty
- Modeling outward reinsurance
- Conclusions on designing models

# Introduction

# Introduction

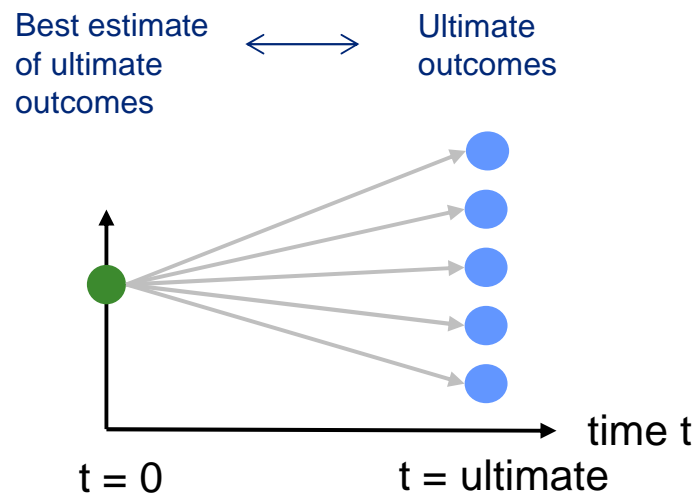
## The one-year risk for non-life business

With the modern Solvency regimes such as the SST and Solvency II, quantifying the one-year risk has become increasingly important

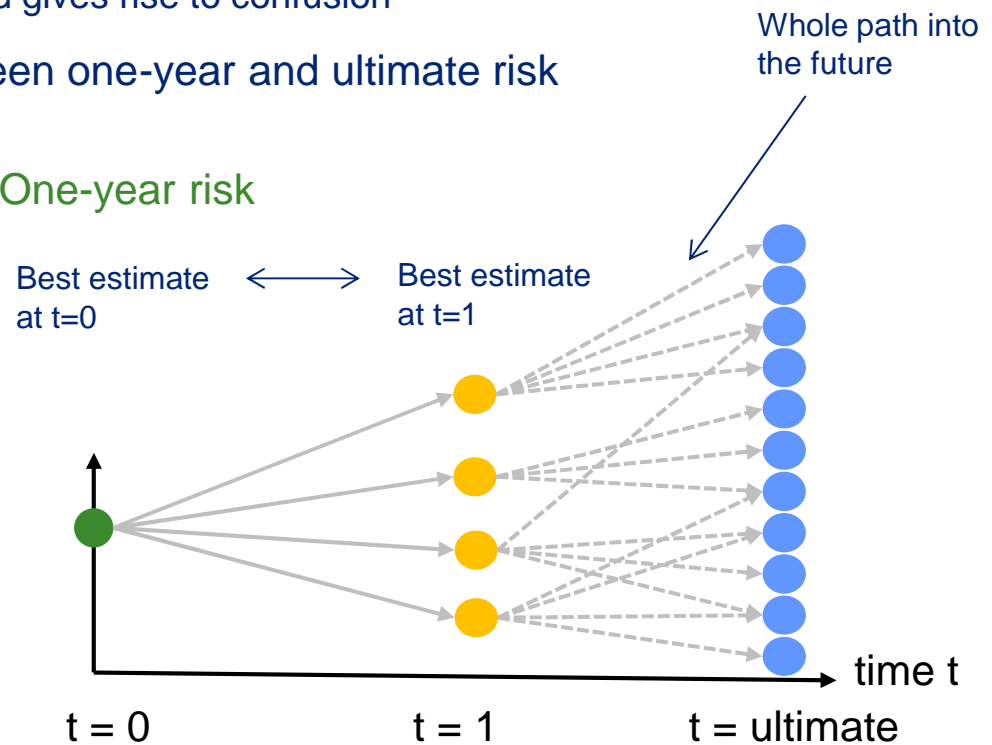
- In non-life insurance modeling the focus has historically been on ultimate risk
- The quantification of the one-year risk remains an on-going challenge
- One-year risk is not easy to understand and gives rise to confusion

There are fundamental differences between one-year and ultimate risk

### Ultimate risk



### One-year risk



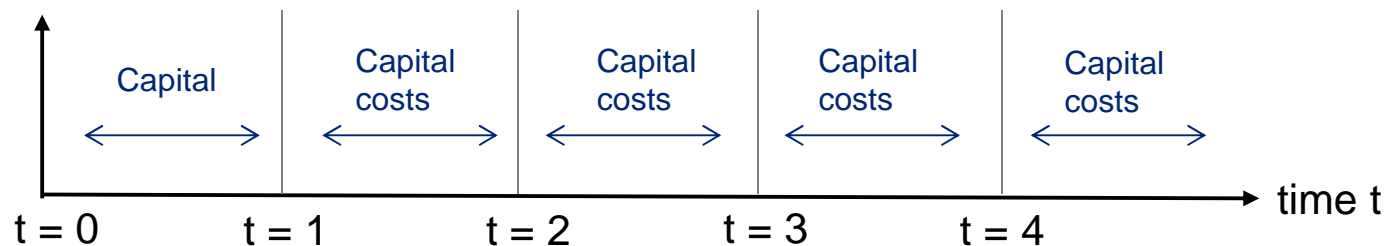
# Introduction

## Do modern solvency regimes only consider the one-year risk?

The primary objective of solvency regimes is the protection of policyholders

- Policyholders must be protected not only over one year but to ultimate  
→ Should one not look at the ultimate risk rather than the one-year risk?

How is protection over an ultimate horizon achieved e.g. in the SST and Solvency II?



- At time  $t=0$ , there needs to be sufficient capital for the first year (time  $t=0$  to  $t=1$ )  
→ One-year risk capital, SCR
- There needs to be sufficient capital costs for all years after the first year  
→ Risk margin, MVM

Note that this implies that the company might need to raise capital at the end of every year!

# Introduction

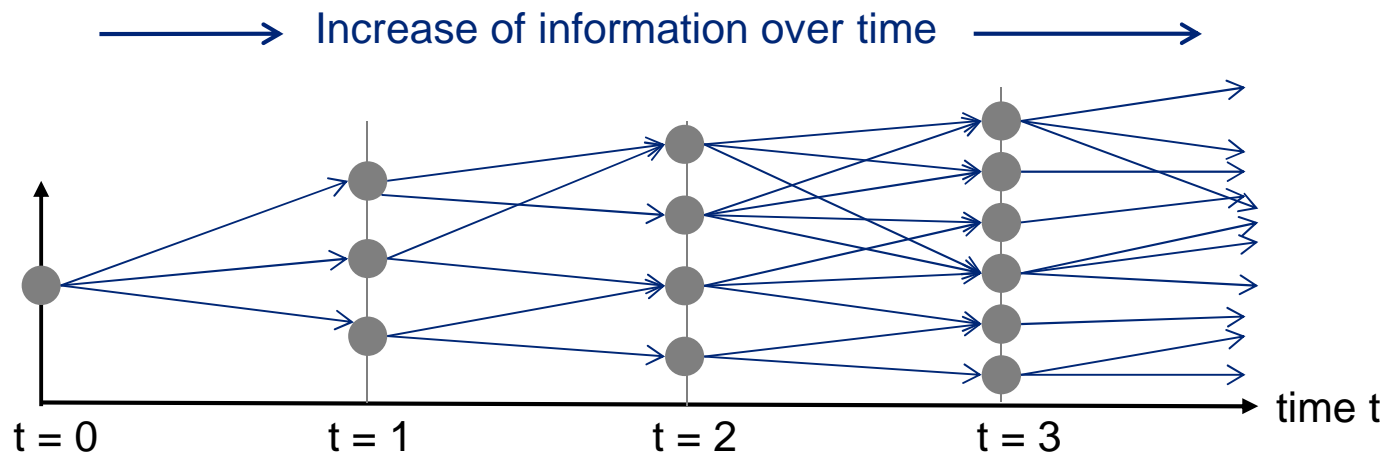
## Gain of information over time

For the one-year risk as opposed to the ultimate risk, one does not need a model for the ultimate outcomes but a model for how information is gained over the first year (and later years for the risk margin)

- One possibility: one-year risk = ultimate risk plus model for gain of information over first year

Modeling this gain of information over one year is one of the main “new” challenges for one-year risk models

- Interestingly, reserve (risk) models such as Mack’s chain ladder already implicitly contain the necessary assumptions



# Decomposition of the one-year risk

# Decomposition of the one-year risk

## The overall view

For the calculation of the required capital, the relevant quantity is the one-year change in the risk-bearing capital RBC (or basic own funds), equal to assets minus liabilities, i.e.

$$RBC_1 - RBC_0$$

For the non-life insurance risk, we need to consider business already written as well as business planned to be written in the current year. The relevant quantity is thus:

$$BEL_0 - BEL_1 + Prem^{cUWY}_1$$

- $BEL_t$  = Best estimate at time t=0,1 of ultimate claims for the business written at time t=0,1
- $Prem^{cUWY}_1$  = Best estimate at t=1 of written premiums for current underwriting year (addition to the balance sheet from t=0 to t=1)

Note: Definition of “contract boundaries”; above we disregard changes in interest rates, FX rates etc. as well as outwards reinsurance

In practice, the above expression is typically decomposed into a sum of terms, where the individual terms are modeled by dedicated models (e.g. premium risk, reserve risk)



# Decomposition of the one-year risk

## Decomposition into different dedicated models

The non-life insurance risk is typically modeled by dedicated models for a decomposition of

$$BEL_0 - BEL_1 + Prem^{cUWY}_1$$

Why?

- *Business that is written and earned at t=0*: no claims can “occur” between t=0 and t=1; best estimate reserves may change due to new information
- *Business that is written and not earned at t=0*: claims can occur between t=0 and t=1; there is claims experience at t=0 from business from the same underwriting year
- *Business that is not written at t=0 and earned at t=1*: claims can occur between t=0 and t=1; no claims can occur after t=1
- *Business that is not written at t=0 and not earned at t=1*: claims can occur between t=0 and t=1; claims can occur after t=1

So there is the possibility for several individual models

- Often, one distinguishes premium and reserve risk, separate by prior and current underwriting or accident year
- Additional models might be required

# Decomposition of the one-year risk

## Common market practice and comments

Recall one-year risk:

$$BEL_0 - BEL_1 + Prem^{cUWY}_1$$

One possible decomposition is as the sum of the following components:

Reserve risk	$BEL^{pAY}_0 - BEL^{pAY}_1$	<ul style="list-style-type: none"> <li>• Risk of the business from prior accident years</li> </ul>
Premium risk	$BEL^{cAY}_0 - BEL^{cAY}_1$	<ul style="list-style-type: none"> <li>• Risk of the business from current accident year</li> </ul>
“UPR risk”	$BEL^{WnE1}_0 - BEL^{WnE1}_1$	<ul style="list-style-type: none"> <li>• Risk of the business that is written but not earned at t=1</li> <li>• Not always covered by models</li> </ul>
“Premium uncertainty”	$Prem^{cUWY}_1 - Prem^{cUWY}_0$	<ul style="list-style-type: none"> <li>• Difference in premium expectation for the business to be written between t=0 and t=1</li> <li>• Not always covered by models</li> </ul>
“Expected result”	$Prem^{cUWY}_0 - BEL^{cUWY}_0$	<ul style="list-style-type: none"> <li>• Expected profit from current underwriting year business</li> <li>• Not a “risk” but a summand</li> </ul>

Reserve risk

# One-year reserve risk

## Common market practice

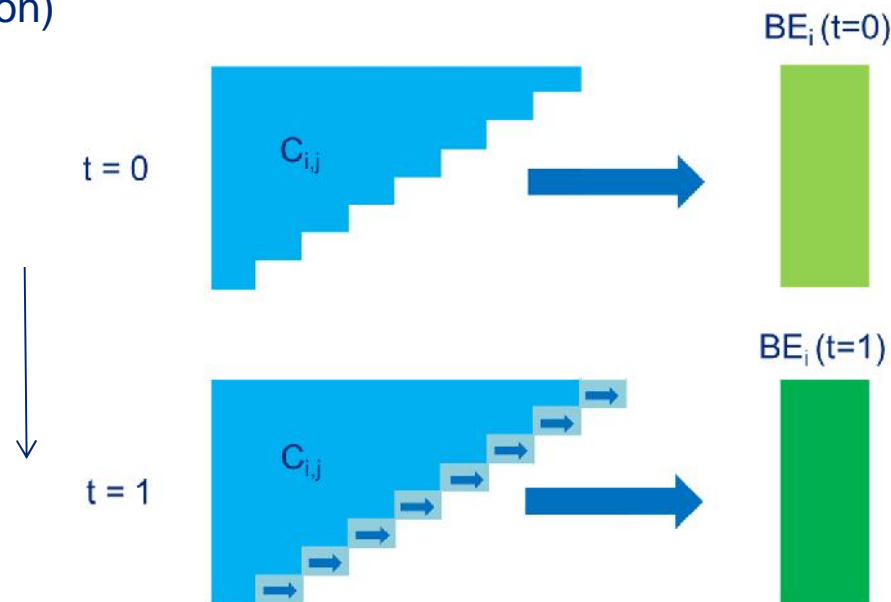
- Common approaches to model ultimate or one-year reserve risk we see in the market can be divided into:

- a) Analytical approaches (e.g. Mack)
- b) Bootstrap approaches

- One-year risk models can be also divided into:

Those that model ultimate risk (perhaps with a transformation to one-year risk in a second step)

Those that model one-year risk directly (e.g. Merz-Wüthrich; more generally with “actuary in the box” assumption)



# One-year reserve risk

## Some considerations

- 1) Appropriate real world assumptions (e.g. about claims reporting and payment process)
- 2) Communication of results – some thoughts on the focus on the CoV
- 3) Consistency of best estimate (BE) calculations for the “available capital” (economic balance sheet) with the risk model
- 4) Consistency within the risk model (between the BE calculations and the production of the new diagonal)
- 5) Consistency between ultimate and one-year risk
- 6) Out-of-triangle information and one-year risk

We discuss each of these requirements in the following

# One-year reserve risk

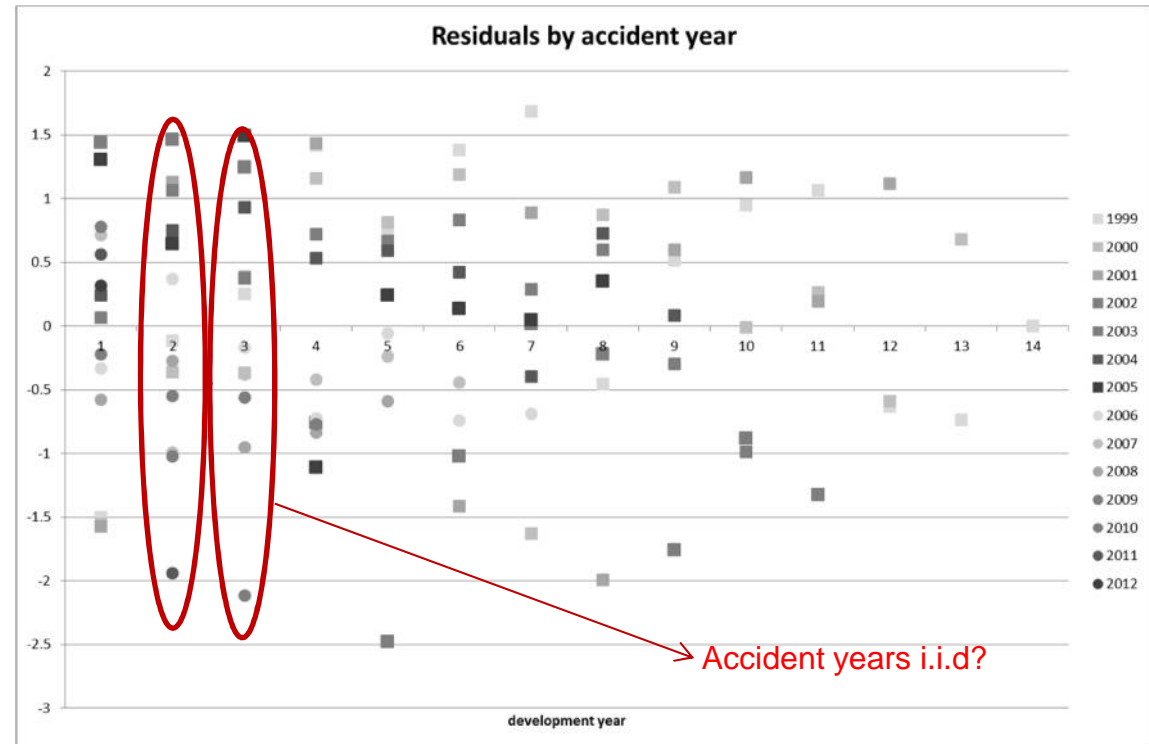
## 1) Appropriate real world assumptions

- All reserve risk models are based on specific assumptions about the real world, e.g. about the claims reporting and payment process.
- For triangular approaches, examples of real world assumptions are (where  $C_{k,j}$  denotes the cumulative paid or reported claims for accident year  $k$  and development year  $j$ ):
  - Chain ladder method assumes that  $C_{k,j+1} - C_{k,j}$  is a function of  $C_{k,j}$   
T. Mack: Distribution-free Calculation of the Standard Error of CL Reserve Estimates. ASTIN Bulletin 23 (1993), 213-225
  - The common assumption of BHF is that  $C_{k,j+1} - C_{k,j}$  is independent of  $C_{k,j}$   
[T. Mack: The Prediction Error of Bornhuetter/Ferguson. ASTIN Bulletin 38 (2008), 87-103.], [A. Saluz, A. Gisler, M. Wüthrich: Development Pattern and Prediction Error for the Stochastic BHF Claims Reserving Method, ASTIN Bulletin 41(2) (2011), 279-313]
- In particular, Mack's chain ladder and BHF are not consistent because they are not based on consistent (and indeed to some extent contrary) real world assumptions  
Nonetheless, it is in our experience common that chain ladder development patterns are used for BHF, leading to a model that is not internally consistent

# One-year reserve risk

## 1) Appropriate real world assumptions

- In practice it is often not explicitly tested whether the real world assumptions on which the used methods are based are valid
- Sometimes, the real world assumptions are not even explicitly known
- It is often argued that expert judgment (e.g. the approach used for parameterisation, and/or adjustment/blending of results) implicitly allows for deviances from real world assumptions.
  - Because the explicit testing and adjustments would be too complicated?



- Whatever the model used, it may make sense to think of scenarios (e.g. inflation, change in law, subrogation for WC – uncertainty around % of liability) when parameterising or validating the model
  - In some cases when data is scanty, e.g. small run-off portfolios, thinking of scenarios using expert judgment may be the only option available in parameterisation

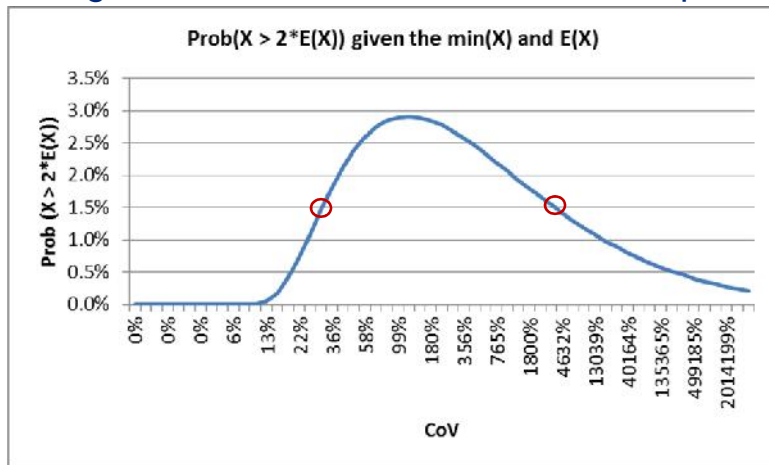
# One-year reserve risk

## 2) Communication of results – some thoughts on the focus on the CoV

Often communication of results within technical teams is done using coefficients of variation (CoVs)

It may be more meaningful and more intuitive to communicate results in terms of quantiles (e.g. what is the probability of the incurred doubling?)

- Often (non-shifted) lognormal distributions are fitted to mean and stdev, which start at zero even when zero ultimate payments is not possible! → consider using shifted lognormal instead
- Assume that we parameterize a shifted lognormal distribution using quantiles. In this case, it turns out that, given the minimum, mean and one quantile (e.g. probability of doubling), there are two solutions



If  $\text{prob}(X > 2 \cdot E(X)) = 1.5\%$ , at first thought I would choose  $\text{CoV} = 30\%$  rather than  $\text{CoV} = 3'000\%$ , but for individual claims in a long-tailed line, maybe the latter  $\text{CoV}$  is more appropriate, where change in information in one-year is very unlikely but changes when they happen are very large.

There are two solutions also for the Gamma, Weibull and Inverse Gaussian

There is only one solution for the Pareto Type I



# One-year reserve risk

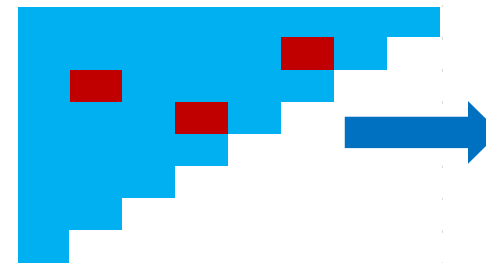
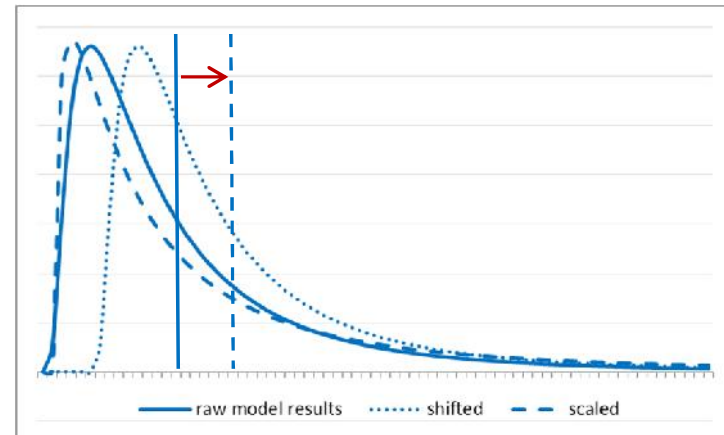
## 3) Consistency of BE calculations for the “available capital” with the risk model

- This is easy if the BEs for “available capital” are calculated mechanically with no expert judgment, e.g.
  - Application of the plain chain ladder method to triangles
  - Application of several methods to claims data with concrete rules for selecting assumptions and blending the results, for example:
    - BHF for the most recent X accident years with a concrete rule for calculation of the a-priori loss ratio from policy & claims data, chain ladder for the prior accident years,
    - use of incurred data for most recent x AYs and paid data for older AYs
- BUT, in practice, BE calculations usually contain assumptions using expert judgment that change from one model calculation to the next.
- Such use of expert judgment is necessary for example when:
  - Assumptions of the available actuarial reserving methods do not hold (e.g. wrt to chain ladder, the claims development is not i.i.d. across accident years)
  - There exists relevant and important information that is not contained in the claims data, e.g. in the “tail”
- AND usually such assumptions are not determined by separate models (e.g. using market data) but by “gut feeling”.

# One-year reserve risk

## 3) Consistency of BE calculations for the “available capital” with the risk model

- The more such expert judgment is used for the BE for the “available capital”, the less appropriate analytical methods are for the risk model.
- The common approaches to allow for this weakness include the following (and each come with certain potentially serious issues):
  - Scaling or shifting of results
    - Based on the difference between the BE and the mean of modelled distribution implied by the analytical risk model
  - Separate allowance for atypical claims
    - Removal of such claims from the triangle used for the analytical method, with a separate allowance for the reserve risk relating to atypical claims – usually ignored!
  - Smoothed triangles
    - Using adjusted triangles for risk calculations
  - Manual overwriting of parameters calculated within the method
    - Could result in material internal inconsistencies and meaningless results
  - Blending of different models, e.g. with paid data & with incurred data
    - OK only if blending approach is entirely consistent with any blending done in BE calculation (usually not possible)



# One-year reserve risk

## 3) Consistency of BE calculations for the “available capital” with the risk model

- The bootstrap models are often more flexible than analytical models and do better wrt to this requirement.
- In theory, the BE methodology can be replicated exactly in a bootstrap model excluding any assumptions that are made with expert judgment.
- It is common for what cannot be replicated within the bootstrap model to be allowed for outside the model by:
  - Scaling or shifting of results
  - Separate allowance for atypical claims
  - Blending of different models, e.g. with paid data & with incurred data

# One-year reserve risk

## 4) Consistency within the risk model (i.e. between BE calculation at t=0, production of the new diagonal, BE calculation at t=1)

- Models/assumptions made for  $BE_0$  implicitly or explicitly contain assumptions about how claims costs will develop in the future. Examples from mechanical approaches are:
  - Chain ladder method assumes that  $C_{k,j+1} - C_{k,j}$  is a function of  $C_{k,j}$
  - The common assumption of BHF is that  $C_{k,j+1} - C_{k,j}$  is independent of  $C_{k,j}$
- MW method has this consistency
- It is important for the bootstrap models to ensure that the projection of future diagonals is consistent with these assumptions
- This is challenging when the BE methodology is not mechanical and not mathematically tractable. For example:
  - If a development factor for BE is selected using expert judgment
  - No clear rules about how the a-priori loss ratios are updated
  - “Manual” adjustments to projected ultimates
- Bootstrap models often need/make simplifying assumptions in addition to real world assumptions made for BEs

# One-year reserve risk

## 5) Consistency between one-year and ultimate risk

- This consideration is relevant when the “ultimate” risk is determined first, and the one-year risk determined from “ultimate” risk
- Such an indirect approach may be preferred for portfolios where influence of *out-of-triangle information* is significant (e.g. changes in external environment, non-proportional claims below threshold, complex reinsurance programme), thus it is better to focus first on getting the ultimate risk right, followed by a simple approach to move from ultimate to one-year (rather than trying to build a very complex one-year model with the so-called “actuary in a box” assumption)
- White & Margetts in their GIRO presentation (2010) discuss 5 possible methods to move from “ultimate” to one-year reserve risk distributions
  - Run off reserve risk using loss pattern
  - Independent, Normal distributed stochastic incrementals
  - Independent, Poisson distributed stochastic incrementals
  - Time-scaling
  - Normal distributed stochastic development factors
- All of these approaches are proxies and none of them is shown to be consistent to the ultimate risk model. The simplifying assumptions made should be as consistent as possible assumptions made when projecting ultimate risk (e.g. about the correlation of  $C_{k,j}$ )

# One-year reserve risk

## 6) Out-of-triangle information and one-year risk

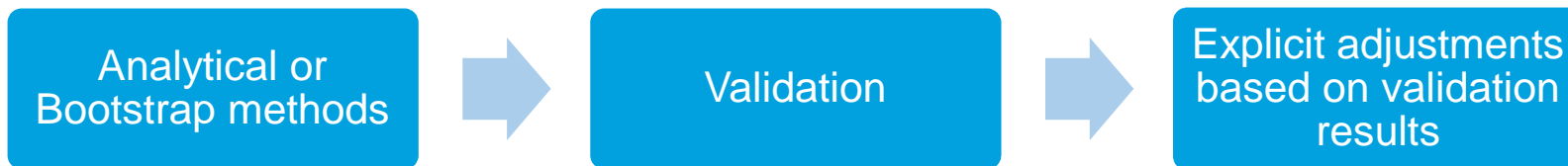
Ultimate risk and one-year risk may behave very differently, generally where relevant losses take many years to be recognised, but in particular where *out-of-triangle information* is material. Consider the following examples:

- A long tailed line and a significant part of the ultimate uncertainty relates to out-of-triangle information that will be known but not reflected in triangles by the end of the next year
  - Approach using triangles only may underestimate one-year risk
- A long tailed line and a significant part of the ultimate uncertainty relates to a court case that will be decided two years from now
  - This uncertainty should not flow into one year risk
- A non-proportional line and a significant part of the ultimate uncertainty relates to claims that will remain under the deductible in the next year
  - This uncertainty should not flow into one year risk

# One-year reserve risk

## Conclusion

- In most cases, analytical or bootstrap methods will not be able to capture the real world assumptions made for the BE calculation for the economic balance sheet accurately enough and will have some consistency issues
- In these cases, the most realistic results may be possible by taking results of these methods as points of reference and complimenting by a strong validation process



- Validation in particular by scenario analysis and looking at historical CDRs (where available) is important

Premium risk



# One-year premium risk

## Introduction

Premium risk is the risk related to “new” business, i.e. business that between  $t=0$  and  $t=1$  is either written (current underwriting year) or earned (current accident year). Premium risk modeling is similar to actuarial pricing, but on a one-year basis instead of an ultimate basis.

A main reason for modeling premium risk by a separate model from reserve risk is:

- Claims can “occur” for the business between  $t=0$  and  $t=1$  (e.g. if an earthquake event occurs between  $t=0$  and  $t=1$ , this can lead to a claim)

This suggests that premium risk is qualitatively different from reserve risk and maybe “more risky”

- This point is often addressed by having a separate “large loss model” in addition to an attritional loss model

In our experience, two main challenges for one-year premium risk model are:

- 1) Consistency between premium risk and reserve risk (and within premium risk, between attritional and large loss model) (e.g. claims inflation)
- 2) Quantification of the one-year risk (specifically, consistency between ultimate and one-year risk)

# One-year premium risk

## Market practice

Examples of premium risk models (excluding Nat Cat) used in the market that we know of include:

- 1) *Models of annual aggregate claims*: either for all claims or in only for attritional claims in combination with large loss models, sometimes on loss ratio basis (i.e. based on historical loss ratios), extrapolation from reserve risk model to current accident year
- 2) *Large loss models (frequency-severity) based on large loss experience*: where either the ultimate risk is quantified and assumed to be equal to the one-year risk, or the ultimate risk is scaled to the one-year risk
- 3) *Large loss models (frequency-severity) based on exposure*: e.g. for Property, Credit and Surety, Liability, where either the ultimate risk is quantified and assumed to be equal to the one-year risk, or the ultimate risk is scaled to the one-year risk
- 4) *Risk driver-based models*: which either initially model the ultimate risk, or the risk driver is directly quantified on a one-year basis so that the one-year risk results from the impact of the risk driver on the portfolio under consideration (e.g. claims inflation, Nat Cat models)
- 5) *Models based on pricing*: special case of 2) and 3), where for every individual contract, a distribution for the ultimate risk is available and aggregated

With models that scale from ultimate to one-year risk, the main practical issue is that the ultimate risk and the scaling methods should be consistent, i.e. based on consistent real world assumptions. This is not fulfilled in the models we know.

# One-year premium risk

## Potentially desirable features

We are not aware of a “perfect” premium risk model in the market. The following aspects can be relevant for premium risk models:

### 1) Use of exposure information (about portfolio and also about underlying events)

Many models are experience-based (actual claims experience) rather than exposure-based

Often, the only direct exposure information used is the historical premium volume – compare this e.g. to (idealized) Nat Cat models

The exposure information is used to “as-if adjust” the historical experience → when appropriate?

Existing exposure models as e.g. used in pricing (exposure curves) are usually for the ultimate risk

### 2) Risk driver-based model instead of sub-portfolio-based model

The structure of typical models consists of sub-models per sub-portfolio (e.g. LoB) together with a copula that is supposed to capture the dependencies between the sub-models

In risk driver-based models, losses result from the impact of risk driver changes on the portfolio (e.g. model for risk driver claims inflation) → e.g. use joint drivers for premium and reserve risk

Justification for dependency assumptions by empirical data is usually not possible, so the justification is often based on risk drivers (as causes of dependency), which means the implicit use of a risk driver-based model

E.g. if A and B are strongly dependent and B and C are strongly dependent, are then also A and C strongly dependent?

# One-year premium risk

## Parameter estimation, as-if adjustments, and real world changes

Many premium risk models we know are based on historical claims experience

- The model parameters are estimated from historical experience data by “as-if adjusting” the data to make it representative of the current situation
- For instance, one might use the historical loss ratios and adjust them for premium rate changes

As-if adjustments are based on usually implicit real world assumptions, which, typically, mathematically, are the following:

- The as-if adjusted data represents i.i.d. samples of the random variable whose distribution we are estimating

This raises the question under which real world changes (e.g. changes in the portfolio) that might have reasonably occurred (e.g. growth or shrinking of the portfolio, premium rate changes, changes in claims inflation) the real world assumption and thus the as-if adjustment method is valid

- The real world changes under which the commonly used as-if adjustments are “correct” are quite limited (example: the distribution of the loss ratios is not in general invariant under “doubling the portfolio” by “adding a copy of the portfolio”)
- There seems to be little awareness of these issues and only few investigations know to us

Parameter uncertainty

# Parameter uncertainty

## Common market practice



Risk models, in our context, are designed to capture the volatility in random real world processes. The results of risk models (e.g. the quantified volatility) are subject to uncertainty:

- 1) Uncertainty in model results due to uncertainty about parameters estimated from limited amount of real world observations (let's call this "parameter estimation uncertainty")
- 2) Uncertainty about real world assumptions on which the model is based ("model risk")

Parameter estimation uncertainty results if the real world observations together with the real world assumptions are not sufficient to uniquely determine the model parameters. To quantify parameter estimation uncertainty requires assumptions about "synthetic real world observations".

Common market practice:

- 1) is often, at least partially, quantified and included in model results
- 2) is often not quantified, sometimes captured via sensitivity testing

# Parameter uncertainty

## Parameter estimation uncertainty, important considerations

Parameter estimation uncertainty results from using a limited volume of relevant data for the model calculation

- It is common to represent the total risk as the sum of
  - “Process risk”, the risk if the model parameters are assumed to be uniquely determined by the real world observations
  - “Parameter risk”, the risk coming from the uncertainty about the estimated parameters if process risk is “turned off”

In the typical case, the results of models that consider parameter estimation uncertainty correspond to combination of process risk and parameter risk as above
- The more general definition of parameter estimation uncertainty is the uncertainty in the model results given the real world assumptions, reflecting the amount of real world observations used. This avoids the “parameter risk of parameter risk” problem
- Parameter estimation uncertainty is sometimes confused with uncertainty caused by regime changes or seasonality in the real world. If this is not explicitly covered by the model, it would be “model risk”. If it is explicitly covered, it is just part of the model

Parameter estimation uncertainty should be quantified for transparency reasons and included in the model for consistency reasons

- This does not imply that a model that includes parameter estimation uncertainty produces “better” model results than a model that does not
- There is additionally “model risk”, i.e. the uncertainty in real world assumptions
- It is important to estimate “model risk” through applying and reporting sensitivity tests

# Modeling outward reinsurance



# Modeling outward reinsurance

## Common market practice

- BE reserves: It is worth noting that in calculation of even just the best estimates of recoveries from non-proportional contracts (i.e. before one-year risk), the uncertainty in ultimates (“reserve risk”) needs to be considered

E.g. when negotiating commutations

Allocating recoveries to different layers (these may in turn trigger reviews by claim managers)

For non-proportional reinsurance, it is incorrect to take gross BE minus retention as the estimate of the recovery. One needs to allow for claims now under the retention but may reach the retention, and vice versa.

For example, if the gross best estimate of a claims is 90, and a per-risk XoL reinsurance contracts starts at 100, the net < 90 as long as the claim is open and there is some chance that it may exceed 100. This is obvious but not always considered in practice when making point estimates.

Analytical solutions not always possible

numerical approaches may be needed for some distributions for exceedance functions (e.g. gamma)

- Premium risk: common market practice is to model large claims individually and non-proportional reinsurance accurately by contract, at least on an ultimate basis

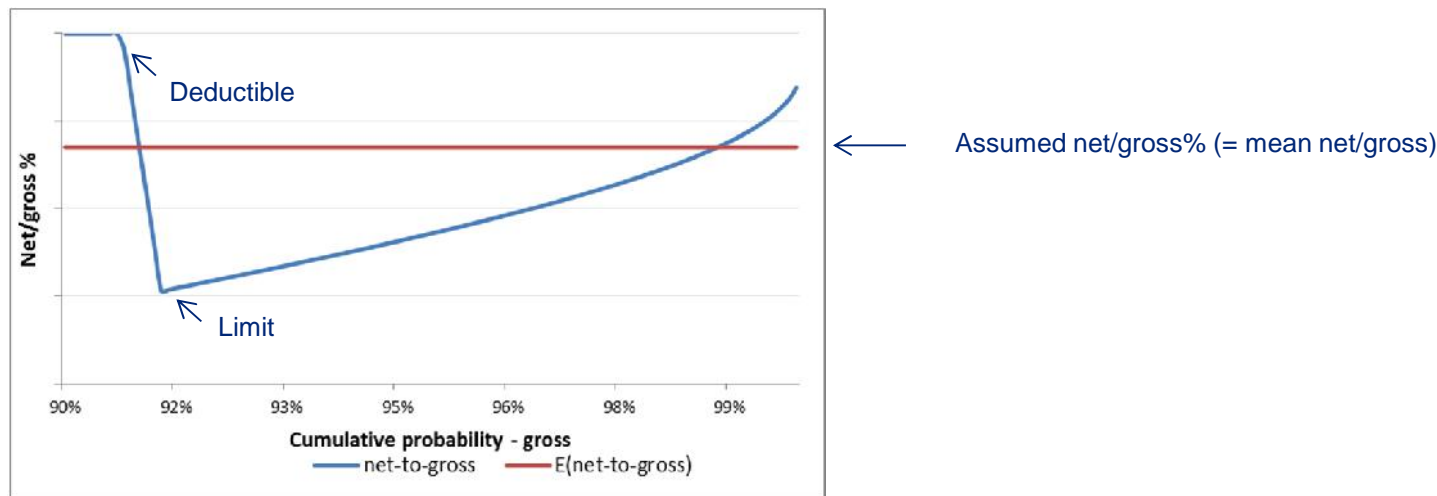
Some minor simplifications may be necessary (e.g. for reinstatements) for operational reasons

# Modeling outward reinsurance

## Common market practice

- Reserve risk: a common approach is to model gross claims in aggregate by accident or underwriting year and net claims with the “virtual quota-share” assumption

This may result in material errors for some non-proportional contracts



To understand the potential materiality of the error, it is necessary to think about policy limits, retention/deductibles and limits of the reinsurance covers, and the large claim severity distribution

An alternative approach is to model gross and net separately (problematic when reinsurance has changed over time) and specify a dependency, typically fully positive

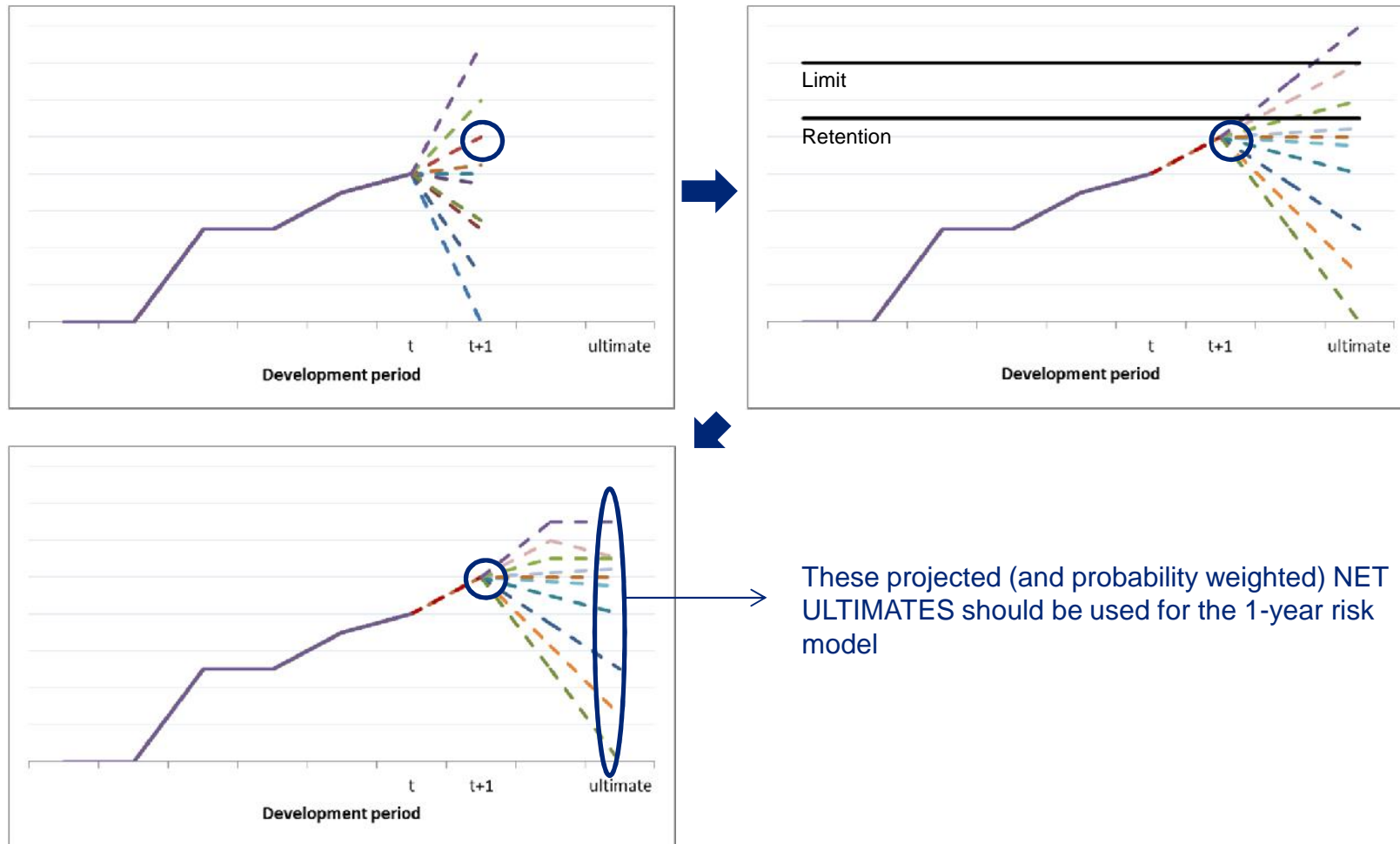
Accurate modelling of per-risk non-proportional reinsurance requires separate modelling of large claims (above retention/deductible)

- Need to weigh up benefits against increased workload, challenges in parameterisation (principle of parsimony)

# Modeling outward reinsurance

## Consideration for one-year risk

- Non-proportional reinsurance: cannot apply structure to year-end best estimates needs projection to ultimates

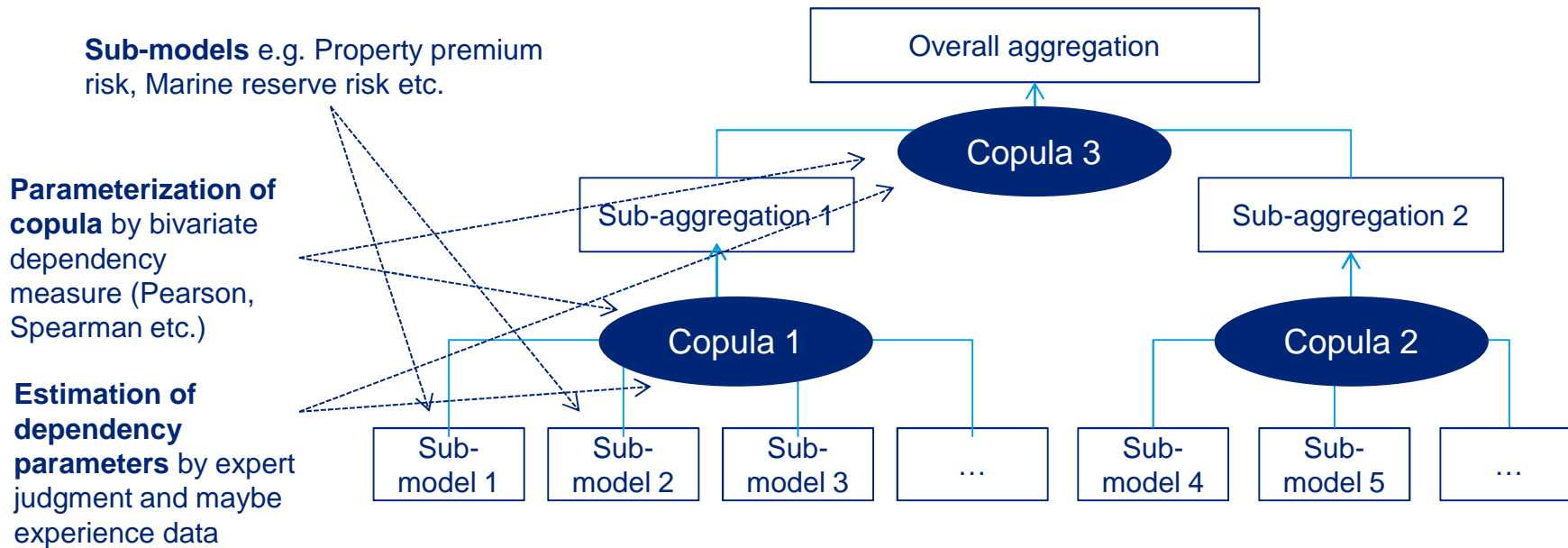


# Conclusions on designing models

# Conclusions on designing models

## Common market practice

The high-level model structure of the default model for (one-year) non-life insurance risk (excl. Nat Cat) looks as follows

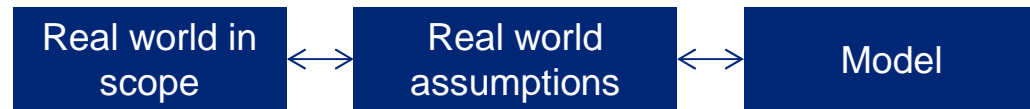


Questions:

- Is this a “good” model?
- Are there “better” alternatives?

# Conclusions

## What is a good model?



It is a fact of our life that we often do not know whether the results of our models (e.g. the 1% expected shortfall or the 99.5% VaR) are correct. So the quality of a model can only to a limited extent be assessed on the basis of the results the model produces.

The following model quality criteria are very important in our view:

- **Internal consistency:** Identification of the **real world assumptions** on which the model is based, justification of the assumptions, and demonstration that they are consistent, i.e. do not contradict each other (see e.g. Solvency II draft Level 2 IM Art 219 TSIM9.(2))

The **real world assumptions** above are the assumptions about the real world under which the model describes the real world correctly. So the requirement essentially means that it must be known under which assumptions about the real world the model is correct, these assumptions must be shown to be reasonable, and it must be possible that the assumptions are true in the real world

- **Functional behavior under real world changes:** Comparison of the qualitative behavior of the model outputs under real world changes with the behavior in the real world (see e.g. Solvency II draft Level 2 IM Art 218 TSIM8.(2))

For instance, behavior of the model outputs under changes in the portfolio in scope

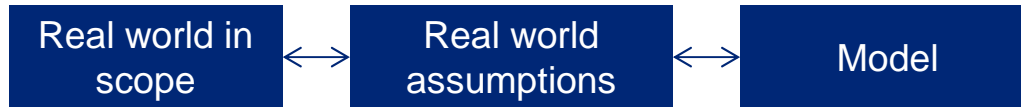
Does the model capture real world functional relationships/ causalities?

Under which real world changes does the model remain appropriate?

Can be model be used for monitoring risk over time?

# Conclusions

## Internal consistency



### Comments on internal consistency

- The real world assumptions on which the model is based are not always known and sometimes have only recently been proposed

BHF assumptions postulated by Mack in 2008

On which real world assumptions is the lognormal distribution based?

- In dependency modeling, what are the real world assumptions underlying a specific copula?

E.g. for Gauss, the Cholesky decomposition provides a functional dependency model

The parameterized copula contains implicit real world assumptions about how bivariate dependencies translate to multivariate dependencies

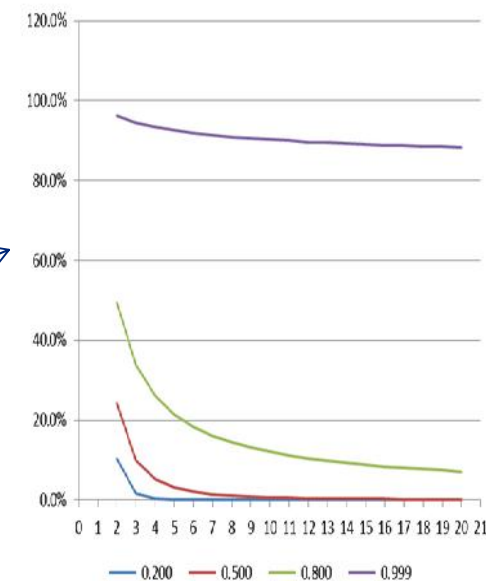
- It is challenging to ensure that the real world assumptions are consistent

E.g. use of chain ladder development pattern for Bornhuetter-Ferguson best estimate

Between premium and reserve risk, between one-year and ultimate risk

- How important is it that a model is internally consistent?

$$\begin{aligned}
 X_1 &= Y_1 \\
 X_2 &= a_{2,1} \cdot Y_1 + a_{2,2} \cdot Y_2 \\
 X_3 &= a_{3,1} \cdot Y_1 + a_{3,2} \cdot Y_2 + a_{3,3} \cdot Y_3 \\
 &\text{etc.}
 \end{aligned}$$



Implied conditional exceedance probability at the 95% quantile for a bivariate normal distribution for different correlation numbers

# Conclusions

## Functional behavior under real world assumptions

### Comments:

- The default non-life insurance risk model structure typically captures real world relationships neither in the sub-models (relationships about how a risk driver causes a claim for the sub-model), nor in the aggregation of sub-models (causal dependencies, common risk drivers)
- One can show with easy examples that the dependencies between sub-models are not independent of the portfolio underlying the sub-models. This means that one cannot separate the parameterization of marginals from the parameterization of the copulas
  - Dependency parameter estimation by expert judgment should also consider this fact
- Functional behavior under real world changes is relevant for the estimation of model parameters in models based on experience data, because the historical experience data needs to be as-if adjusted to make the adjusted data representative to the current situation
  - Typical as-if adjustment methods are valid only for a very limited range of real world changes
- In a risk driver-based model, functional relationships can be explicitly captured in the model
  - Risk factors and exposure functions and functional dependencies instead of marginals and copulas
- The model design should start with an overall high-level model design



**Thank you**

**Questions?**

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